This is a slightly modified/updated version of the *Constructionism 2010* paper.

Restructuring Change, Interpreting Changes: The *DeltaTick* Modeling and Analysis Toolkit

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Abstract

Understanding how and why systems change over time is a powerful way to make sense of and navigate our world. By modeling those systems, learners have the opportunity to consider how their own actions influence the world, and to make predictions and recommendations for the future. But often, the very notion of change is as complex as it is powerful - population levels, global temperatures, or economic trends are all driven by *multiple* events and actors, but measured in terms of only a few quantities. In this paper, we discuss the motivation, design, and pilot user studies of *DeltaTick*, an extension to the NetLogo (Wilensky, 1999) agent-based modeling environment that allows students to easily build and analyze sophisticated models of quantitative change within specific real-world domains. To do so, they construct models in terms of *agent behavior-based* units, rather than the *rate-based* units typical of calculus-based or system dynamics models. They can then explore those models with specific attention to plots of quantitative trends that result, providing multiple opportunities for them to connect and compare their behavioral models to typical equation-based representations.

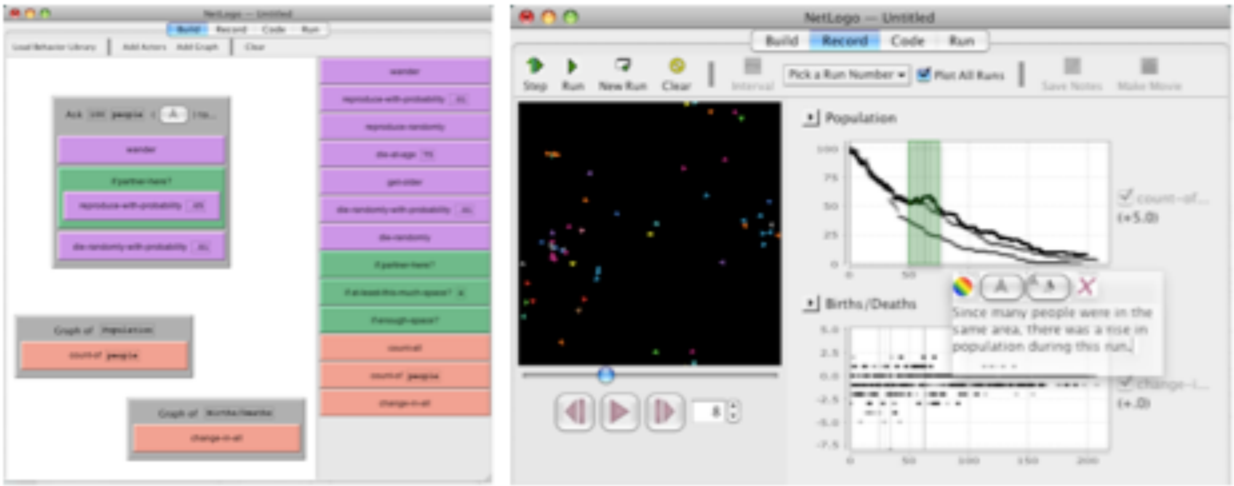

Figure 1. Model building (left) and analysis (right) in the DeltaTick Environment

This environment leverages what Wilensky and Papert (2006; In Prep.) refer to as a *restructuration*: a re-encoding of disciplinary content with a new representational technology. Restructurations can exhibit different properties: for the purposes of this paper, we focus on how learners' construction and interaction with agent-based representations of change can help emphasize (1) the *relevance* of ideas of change and variation to students' own experiences as actors and observers in their world, and (2) the *learnability* of some core underlying concepts of change and variation. In addition to implementing a constructionist paradigm (Papert, 1980), our design and activities integrate perspectives from complex systems education (Wilensky & Resnick, 1999), low-threshold agent-based modeling environments (Kahn, 2007; Repenning & Ambach, 1997; Wilensky, 2003), representational infrastructure shift (diSessa, 2001; Kaput et al, 2002) and intuitive calculus (Kaput, 1994; Nemirovsky et al, 1993; Stroup, 2002).

Keywords

restructuration, mathematical modeling, agent-based modeling, scientific literacy

Restructuring Change

We want to give students a way to critically think about how they influence and are influenced by large-scale, systemic changes in their world. *DeltaTick* is a simple, extensible construction and analysis toolkit to support this goal, leveraging recent findings regarding how people think and learn about quantitative change. In this paper, we describe the motivation and design of this environment, and discuss preliminary results from interviews with students who use it to construct and explore alternative models of population growth. We argue that by providing students with tools and activities that allow them to express notions of rate and accumulation as quantitative outcomes of specific individual behaviors as they occur over time, students can model and explore quantitative change in a way that (a) emphasizes its relevance to their own lives by leveraging their own experiences of actions and change in the world, but that also (b) provides a novel access point to many of the ideas of mathematical change as they exist in more typical calculus and differential equations-based representations (namely, notions of derivative, integral, and the reversibility of the two).

By changing the way that students can express and explore the notions of rate of change and accumulation, DeltaTick serves as an example of Wilensky and Papert's (2006; In Preparation) notion of a *restructuration*: a re-encoding of disciplinary knowledge using a new representational technology. These different "structurations" of information - in this case, using agent behavior rather than systemic rates to explore how a system is changing over time - afford different ways of thinking about and interacting with the same content. This particular agent-based structuration of change provides a new language to "speak" and practice mathematical modeling and the mathematics of change in systems. Furthermore, we argue that similar to the way that computational representations of motion have helped students to access notions of "qualitative calculus" and relate those notions to their own experiences (Stroup, 2002; Kaput, 1994; Nemirovsky et al, 1993), this structuration can be leveraged to provide students with new access points to the more typical notions of rate and accumulation, but as to relate to a different sort of experience – those that students have as they think about the behavior of components in a system, and as they participate as members of a community, nation, and world.

The theoretical and design contributions of this work are threefold. First, we are leveraging and contributing to existing work on students' experiences and understanding of quantitative change, but in the specific context of change in complex systems, where multiple interactions and events are embedded in only one or a few measured trends. Second, we are exploring a design space that provides students a low-threshold entry point to building flexible, personally relevant scientific models while still having the opportunity (and perhaps even encouraging) more flexible and sophisticated model refinement. Finally, we are exploring the role of tools for the analysis of student artifacts in constructionist environments – such that those tools and students' own artifacts serve as a bridge to typical representations of disciplinary content. In terms of practical contributions, we are working toward providing students with a viable, intellectually honest alternative to symbolic calculus for modeling mathematical change, while at the same time providing a potential bridge to more typical calculus-based concepts. Our goal is to present the mathematics of change as a relevant, accessible, and empowering tool that can help students understand and predict their world.

Motivation

Examining rates of change over time and their accumulations have become some of the most ubiquitous practices in not only the natural and social sciences (AAAS, 1991), but also for navigating modern society (Roschelle et al, 2000; OECD, 2006). Often, however, the quantitative trends used to explore economic, environmental, or social realms reflect large-scale, systemic processes that involve and effect a number of actors and events – so it is not just understanding quantitative change, but also understanding how that change reflects the events and interactions of a given system – that helps us to make sense of the world and our role as citizens within it.

Shifting representational infrastructures - and specifically, computational tools that represent and simulate processes over time - reflect a powerful way of exploring, thinking about, and simulating change over time - and potentially, for allowing more people to do so (Papert, 1980; diSessa, 2001; Kaput et al, 2002). Agent-based modeling (ABM; Langton, 1997; Wilensky & Resnick, 1999) is one example of a computational representation appropriate for modeling complex systems such as those described above. This technology models a phenomenon by encoding the behaviors and interactions of individual agents or elements of a system (for instance, the rules that govern motion and collision of particles in a gas), and then simulating that system by having a collection of those agents execute those behaviors over time (for instance, to illustrate how that gas exerts pressure on a container; Wilensky, 2003). It has fundamentally changed *how* scientific content is represented and explored, as well as *who* can author and interact with that content (Blikstein & Wilensky, 2009; Levy et al, 2004; Sengupta & Wilensky, 2008).

But while building and interacting with agent-based models can help students develop a more deep and generative understanding of traditionally advanced content, less is known about how they link this understanding with the more typical representations of those concepts - namely, algebraic and calculus-based equations. This project explores how educators can leverage ABM to provide access to the mathematics of change and the ways it connects with the mechanisms and phenomena it represents.

In the following sections of this paper, we describe in more detail the notion of *restructuration*, and make the case for how an agent-based modeling restructuration of dynamical systems can provide students with more access points to not only specific scientific content, but also to the calculus-based mathematical representations typically used to present that content. Next, we briefly describe the design of a set of computational tools to provide students the opportunity to build and explore agent-based models with explicit focus on how those models represent mathematical change over time; along with a description of the sort of activities that would give students the opportunities to use these tools meaningfully and constructively. Finally, we discuss some findings from a pilot implementation of earlier versions of these tools. We argue that these findings suggest that constructing and analyzing agent-based models with specific attention to ideas of change and variation in systems helps students to understand the *relevance* of ideas of change and variation in their own lives, as well as makes many difficult concepts in change and variation (such as the reversibility of rate and accumulation) more *learnable* for students. We conclude with a brief discussion of future work and implications.

The Computational Restructuration of Mathematical Modeling

Above we discuss the notion of *restructurations* (Wilensky & Papert, 2006; In Prep.) – that different technologies can encode the same phenomenon in very different aspects ways. One of the most important aspects of restructurations is that they exhibit different properties: some are more efficient and precise, while others can make certain content more accessible and usable. Each can also complement and inform understanding of the other. Below, we use a figure to illustrate how agent-based modeling (and more generally, computational behavior-based simulation) can be viewed as a restructuration of the ideas of change and variation (boxes 1 and 2), and how it can inform more typical rate-based representations by providing an opportunity to *coordinate* the results of each through the plots or numerical results they generate (boxes 2 and 3). Although the figure is informed by work on mathematical modeling (Niss, et al, 2006) in the sense that a "real situation" is distilled and then formalized into some symbolic notation, we heavily adapt it here to reflect that those situations can be differently conceptualized, that different conceptualizations can be more or less commensurate with a given symbolic encoding, and that symbolic encodings can be mathematical or computational. We are also careful to note that while we are highlighting the connections that are emphasized through the activity of modeling, this is not a clean process - connections can be made between or within any world, depending on the similarities recognized and actions taken by the modeler (Pozzi et al, 1996; Noss et al, 1997).

The box to the left represents a "real world" representation or experience of some dynamic system - for instance, trends in unemployment as a student may experience them as he reads a newspaper article, or searches for a summer job. Those experiences and understandings that are viewed as the key elements, events, trends, or patterns for a particular phenomenon of interest (in this case, unemployment) can be considered together as a “situation model”. In the case of unemployment, an individual may think of his own and his friends' experiences in the workplace; but he may also consider a recent history of rising unemployment, or of national or international trends in consumer spending. These different ways of conceptualizing the causes and effects of a changing system are more or less appropriately represented by different symbol systems - agent-based modeling, for instance, is more appropriate for encoding individual experiences and interactions; while a differential equation or system dynamics model is more appropriate for considering larger-scale patterns and historic trends. Each restructuration, however, can generate results that can be compared and coordinated with one another – so that encodings in one structuration (the specific circumstances that lead to an individual getting or losing a job) can be compared to those in another (increases and decreases in employment levels), and the relationships between them interrogated (as more people are hired, they are able to spend money, which in turn allows more companies to hire more employees).

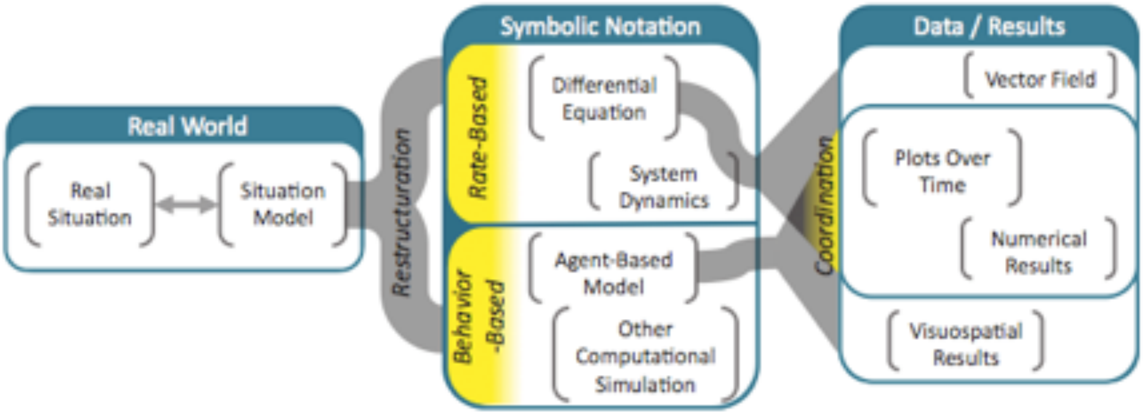

Figure 2. Restructurations are alternate encodings of the same content, with each encoding emphasizing different aspects of that content.

As a result of this shift, agent-based modeling encodes and reflects quantitative change in a way that includes a clear link to specific real-world behaviors that change can represent, as well as emphasizes notions of randomness, sensitivity to local conditions, non-uniform distributions, and other powerful ideas characteristic of *systems* that are not dealt with in traditional calculus. It also provides learners an easy to way to manipulate that encoding in ways they find interesting. On the other hand, this encoding de-emphasizes many of the powerful aspects of typical calculus-based methods, such as the ability to optimize, quickly apply solutions to new and different contexts or scenarios, or quickly compute specific solutions. We argue, and provide evidence below, that it is the transition between different structurations – including the practice of building models in each in order to explore and resolve conflicts between them – which is where a lot of learning can happen around the mathematics of change. In this sense, the plots and numerical results produced by each serve as a *bridging tool* (Abrahamson & Wilensky, 2007) for access to and from typically advanced mathematical and computational concepts.

The DeltaTick Modeling Toolkit

The main goal of our project is to provide students an easy way to create models of how specific real-world systems change, and then analyze those models with specific attention to the one or few quantities that are typically used to represent that change. Below, we describe the DeltaTick modeling toolkit and associated activities with specific reference to one such real-world system, population dynamics.

Constructing Models

To build a model, students begin by defining one or more types of *actors*, a collection of homogeneous entities that all behave similarly. Each actor is represented by a window on the construction screen. They can then add to those actor windows one or more prespecified *behaviors* that will happen for each iteration of simulated time or "tick", as well as *conditions* under which some behaviors happen. Finally, they can add a *plot*, also represented by a window, to the screen and add one or more quantities of interest that they wish the plot to feature. Users have the option to move to an "advanced" version of the model that allows them to view and modify the NetLogo code that underlies the visual representation – which allows them much more flexibility and generativity than the visual language alone.

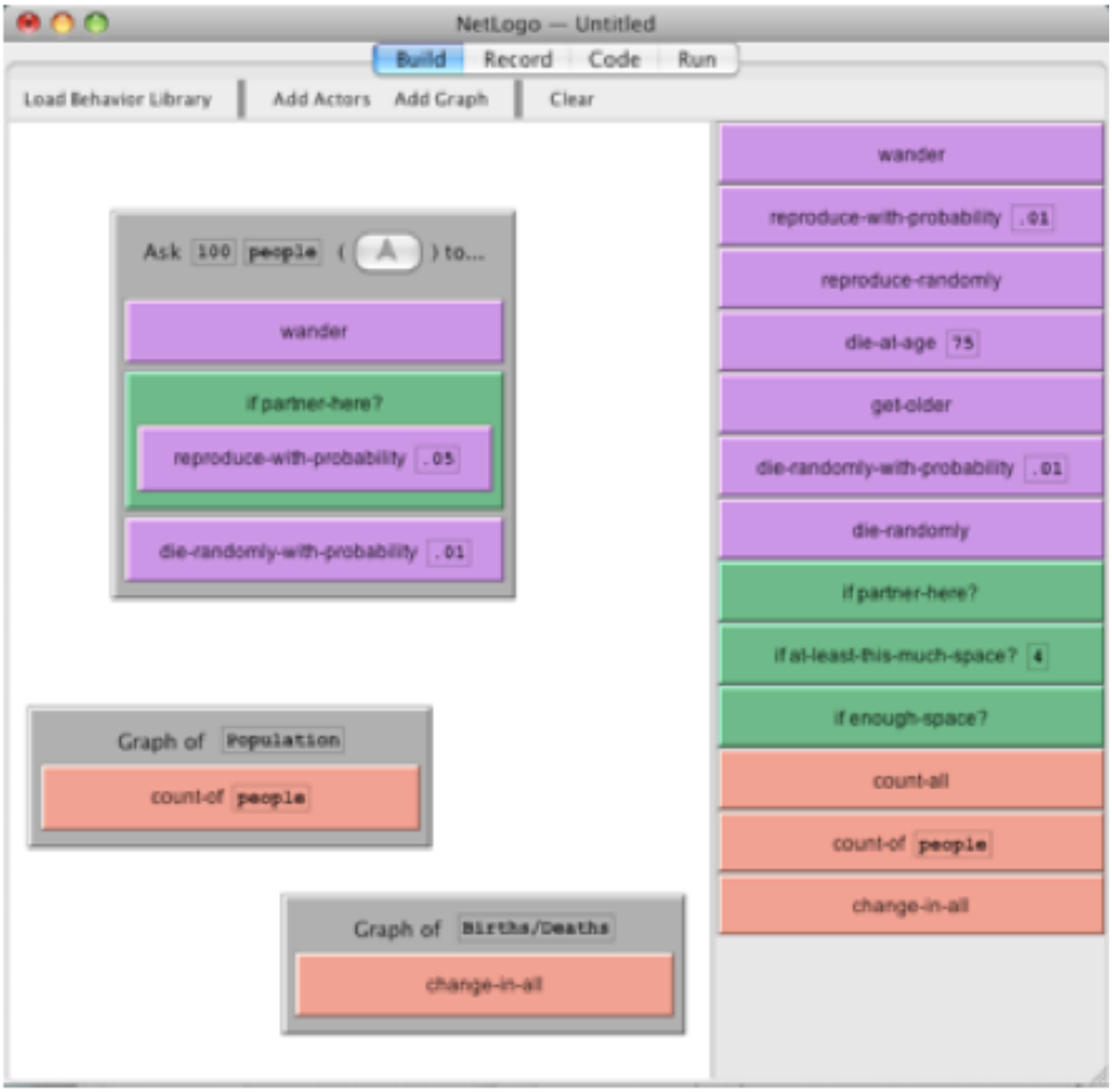

Figure 3. Behaviors are designed to align with different resulting mathematical patterns, as well as allow students to easily move from a custom "template" to directly modifying NetLogo code.

The behavior blocks that are available to students come from a set of domain-specific, extensible "behavior libraries". These libraries are in the same spirit as Kahn's microbehaviors (2007) or Repenning's Behavior Composer (2000) in that behaviors are encapsulated and portable across agents, new behaviors can be written and added to the library, and they can modify pre-existing behaviors before they add them to their model. However, the behaviors in a given library are be designed for specific disciplinary explorations, and in that sense are a bit more specific than the behaviors featured in behavior composer, and larger-grained than microbehaviors. This is intended to preserve a more direct relationship between the addition or removal of each behavior block and changes in the resulting mathematical patterns generated by the model, as well as to provide a considerably "low threshold" access point to model construction.

Analyzing Models

After building a model, students have the opportunity to analyze it using the NetLogo HotLink Replay tool. The two primary representations included in the tool are visuospatial models and time-series plots. These two representations are dynamically linked, such that students can click on critical features of a plot and observe that corresponding point in time in the simulation, or play the simulation over time as a cursor indicates the corresponding area on the plot. Students are able to identify any intervals on a plot, and annotate that interval with respect to the aggregate (whole-system) or agent-based (individual actors) behavior that it reflects. In addition to plots and visuospatial representations, the environment also calculates a user-defined piecewise linear approximation of change on any area of a featured model plot. Finally, HotLink includes an "export movie" feature, which will combine student annotated interpretations of the modeled phenomenon along with the models' visuospatial output to create a captioned replay of the model for sharing.

HotLink Replay is inspired by environments that have enabled students to develop more robust understandings of rate and accumulation by providing them a means to *control* a phenomenon that produces change (Kaput, 1994; Wilhelm & Confrey, 2003); interact with *plots* of change and rate of change over time (Confrey et al 1997); and make linkages between *intervals* and *shapes* of plots and the events they represent (Yerushalmy, 1997).

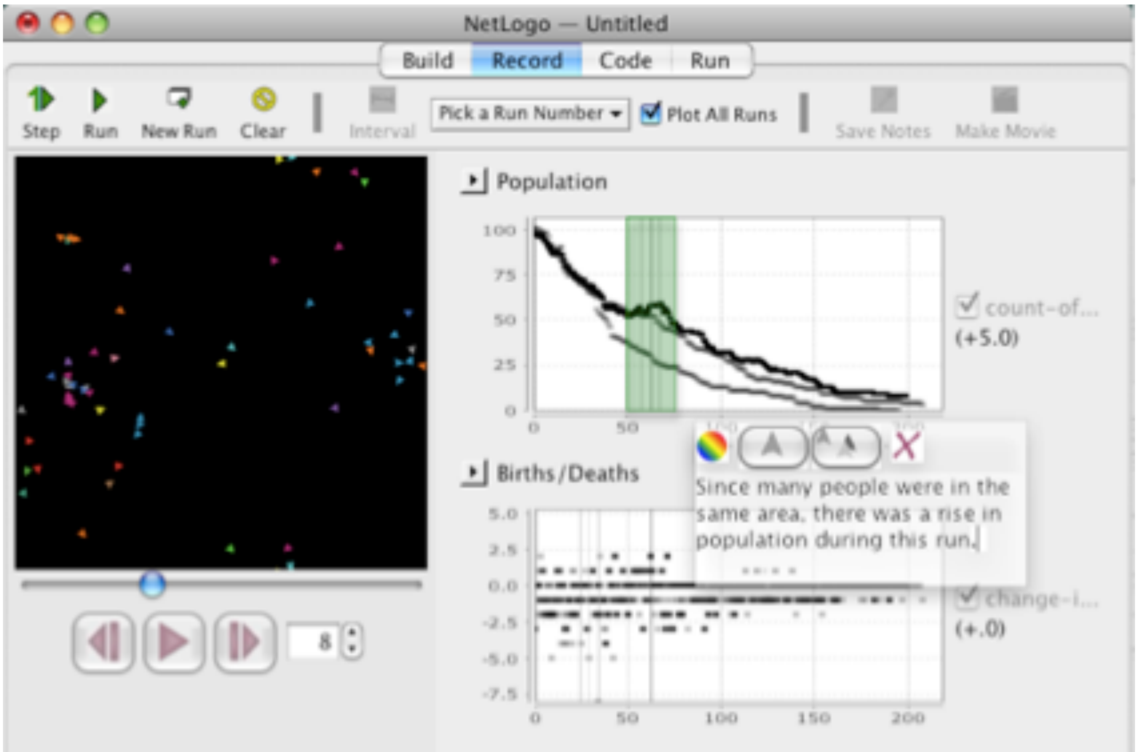

Figure 4. HotLink Replay allows students to replay, annotate, and compare different representations of their models, as well as explore and share interpretations of model results.

DeltaTick Activities

It is not within a tool, but in a student's use of, interaction with, and discourse around tools and activities where learning happens. As such, we argue the our design provides students with the opportunity to engage in the activities, discourse, and kinds of thinking that can help them understand change in their world, view it as relevant to their own lives, and learn the powerful underlying concepts that are so ubiquitous in the natural and social sciences, as well for informed citizenry in an interconnected and changing world. Building on the specific case of population dynamics and informed by pilot interviews and ongoing planning with teachers, we are currently in the process of finalizing and implementing a set of classroom-based modeling activities.

In these activities, students first explore traditional models of population growth by building and analyzing exponential and logistic-like models within the DeltaTick environment. They are encouraged to explicitly draw connections between the mathematical forms of these models and the agent-based models that generate similar patterns. Next, they attempt to "match" specific population patterns (in terms of both shape, and quantity) such as relative stability, extinction, or a sharp exponential (which can occur if reproduction increases as density increases). They are encouraged to compare the models they built to create such patterns. For instance, if some students used only birth and others birth and death to create the same general curve, then they will be encouraged to explore why that is - from this type of activity, students begin to familiarize themselves with the consolidation and interaction of multiple rates to generate a single trend (that is, if the difference between birth and death probability is the same and no other behaviors are included, a similar curve will emerge even with different numerical values). Finally, students are challenged to add conditional behaviors to the model (conditions include space constraints, the presence or absence of other actors, and age-dependence), and make sense of

Once students are comfortable with the modeling environment and ideas of rate of change, behavioral probability, and the role of conditional behaviors in affecting rate in a nonlinear and cross-temporal manner, students are challenged to think of a phenomenon within population dynamics that they find interesting and build their own model. Students will then have the opportunity to model those dynamics, and from those models develop hypotheses about why different patterns of population dynamics look like they do and make recommendations or predictions regarding how they may continue to develop into the future. Students will critique one another's investigations, and eventually complete a research report including an argument for why understanding their phenomenon of choice is important, what the key trends and hypothesis about the phenomenon are, and how their investigation and findings relate to and contrast with those of others in the class.

Results

To explore whether an approach such as the one described above is feasible, as well as to explore our hypothesis that agent-based modeling can provide students a new and productive means to engage with and think about mathematical content, we have used early versions of the DeltaTick tools in clinical interview and classroom settings with U. S. high school students (ages 15-17). Most students were familiar with rate of change and differentials in the context of graphs and tangent lines. In both the interview and classroom activities, students were asked to first interact with a simple model of exponential population growth, where each agent in a system of 100 agents had a 1% chance of reproducing at each "tick", and talk about their understanding of the associated model behavior and graphs at both the agent and aggregate level. Next, they were provided with the "population library" of pre-existing behaviors and conditions available to add to the model, and were asked to change the model as they wished. As students refined their models, in interviews we asked them about their reasoning and expectations for new model and in classroom settings we videotaped their interactions in groups as they built the model; and when students ran the model and analyzed the resulting mathematical trends, we asked them to interpret those trends in both mathematical and behavioral terms.

Below, we argue that in even these short interviews, students (a) recognized that agent-based modeling provided them with an easy way to explore specific, real-world questions about population growth, and that (b) their interactions with the models enabled them to not only better connect mathematical notions of rate and differentiation to the real world, but also to better understand their mathematical meanings.

Relevance: Meaningful Modeling and Extension

One of the questions we were most interested in was whether building models using these behavior-based units allowed students to recognize the relevance of this modeling language and the ideas of change and variation to real-world systems. During our interviews, we encouraged students to modify the model of population growth we initially provided them with in any way they found interesting. When given this opportunity, 7 out of 10 students added behavior parameters or behavior sequences in a way that they explicitly related to real-world behaviors, or included real-world constraints: for instance, students explicitly mentioned issues of life expectancy (*Interview 7*) and the heterogeneity of life expectancy (*Interview 8*), family planning and the likelihood of finding a mate in higher or lower density areas (*Interview 11*), and the role of age in partner selection (*Interview 3*), or the fact that many factors interact to produce patterns in a "nation or city" (*Interview 9*). In classroom settings, students suggested that they would model topics including music marketing campaigns, disease spread, immigration, and natural disasters (such as the oil spill).

In addition to constructing models that they find relevant and applicable for thinking about the real world, we are also interested in making it easier for students to begin to build models that include their own behaviors and code, rather than only using what is available in the pre-specified library. In our interviews, 4 out of 10 students explicitly suggested new behaviors they wished to add the their models that were not available, and 3 of those students actually wrote NetLogo code with the to create new behaviors they could add to their models. This is particularly exciting because interviews were quite short – about 45 minutes each, with only 25-30 of those minutes devoted to model building – and also because none of the students had experience with the NetLogo language. In classrooms, many student groups asked for specific behaviors and capabilities to be added, including differentiation of male and female agents, the potential for multiple births, disease spread, and sterility.

Learnability: Interpreting and Bridging to Conventional Representations

Another question we were interested in was whether students were able to interact with notions of rate of change and accumulation in a way that lets them "unpack" these notions as they relate to change in systems. Yet another is to provide students with alternative representations of change that emphasize important mathematical relations – namely, how a certain rate of change is related to its accumulation over time, and how one can use information about a rate of change to infer a pattern of accumulation. In this section, we recount two such cases.

Rate as Representing Complex Events in the Real World. We found that often in our interviews, making sense of aggregate rates of change in terms of individual behaviors was an interpretive challenge for students (much how it is difficult for students to interpret behaviors at different levels in a systems; Wilensky & Resnick, 1999). In several cases, students started off speaking of rate and derivative as an inert mathematical notion disconnected from the very phenomena it is intended to model. We are interested in the extent to which students begin to link specific quantitative relationships at the individual level to overall systemic patterns. To do so, we are exploring students' ability to quantize the system at four levels: the multiple behaviors of a single agent (individual), the interactions of multiple agents (population), the resulting fluctuations of a single quantity of interest (rate), and the larger pattern of change over time (trend).

We find that students do seem to use the DeltaTick tools to bridge across these multiple levels of reasoning. Here, we recount a case of a group of three students attempting to construct a model in which the population of agents remains relatively steady. Their first attempt results in rapid growth, and their second in extinction. During model refinement, they repeatedly consult their behavior block rules to make sure they are aware of the individual behavior that makes the model run, the visualization to determine whether births and deaths are occurring as they expect, and their graphs to determine general population trends such as stability. In the third model, they reach stability, and one student in the group reasons: "I think basically, like, the fact that they're reproducing at .325, that's basically like one-third. And they're dying every three years, means they're stable, right?"

By observing and controlling a *mathematical* idea in terms of *behavior*, we suggest that this group was able to integrate an understanding of how multiple behaviors interact with one another across individuals, population, and time in order to accurately measure and quantify the results of those behaviors.

Defining Mathematical Terms with Behavioral Relationships. Finally, we argue that representing change in systems in terms of agent behaviors can provide students with insight into how the mathematical notions of rate of change and accumulation relate, and what they represent in a modeling context. In one interview, we ask Brooke (*Interview 3*) how to find the population's rate of change in the first agent-based model. Although the model featured a plot of the number of individuals born at each tick in the model, as well as a plot of the total population per tick from which this information could be extracted, she only suggests that "you could use derivatives" – presumably referring to the mathematical procedure for determining a rate of change given an algebraic expression of the change itself – as a means to determine change for a tick in the model. When probed for whether she could think of any other way "with all the information you're given here", she responded "Um, I dunno, I'd have to think about that. Kind of like derivatives all stuck in my mind."

Later, after interacting more with the model and being asked how the two plots featured in the model are related, she experiences an "aha" moment where she not only recognizes birth in the population as defining the rate of change in this simple model, but also provides a sophisticated description of how notions of rate of change and accumulation relate to the behavior of the model:

S: Um (points at lower graph, then top graph) okay our original population is taking this (points to lower graph) added to uh people that there were, that there were beforehand, (mhm) before they were (mhm) the people were born. Um so it's taking in account to adding to the uh population beforehand (mhm), which is kind of the deal of exponents which is multiplying and multiplying and multiplying (mhm) from the original
M: And so does that help you talk about rate of change at all?
S: In terms of population or in terms of...?
M: In terms of population.
S: People weren't, um, then I guess, oh, I guess this could be the rate of change.
M: And why's that?
S: Uh, because, well this divide, is it, yeah, because their rate of change is saying like oh, well this is how many people were added to the original population over a period of time
M: Mhm and how does that relate to kind of like the ideas you learned in class?
S: Um, about derivatives and stuff?
M: Yeah.
S: Um, that derivatives is basically taking like an exact (mhm) point divided by another exact point finding the exact um, like change, but this gives us the exact change over the exact time. It gives us the exact number of people born at a certain time which is what derivatives is, is solving for.

Discussion and Future Work

So far, the results we have reported are preliminary, and we have changed the design of the DeltaTick environment in part due to lessons learned from the interviews. We are currently finalizing that design, and will soon be conducting a new series of more in-depth interviews and classroom implementations that we hope will provide students more time and meaningful contexts to explore the mathematics of change with agent-based modeling.

Our experiences so far speak to the potential for this sort of approach to provide students with a new way to "speak change", and generate and explore models of quantitative change in systems in personally meaningful and powerful ways. We also believe that by constructing models that express mathematical ideas as the *results* of behaviors, the underlying usefulness and meaning of rate of change – as well as its relationship and reversibility with to accumulation – students have a new access point to understanding those ideas in applicable and conceptually generative ways. Finally, by exploring how different behaviors can all influence one or a few quantities of interest, students can begin to appreciate the complexity and many potential implications carried by information about change in our world.

References

Abrahamson, D., & Wilensky, U. (2007). Learning axes and bridging tools in a technology-based design for statistics. *Int. Journal of Computers for Mathematical Learning, 12*(1), 23-55.

Blikstein, P., & Wilensky, U. (2009). An atom is known by the company it keeps: A constructionist learning environment for materials science using agent-based modeling. *International Journal of Computers for Mathematical Learning, 14*(2), 81-119.

diSessa, A. (2001). *Changing minds: Computers, learning, and literacy*. Cambridge, MA: The MIT Press.

Kahn, K. (2007). *Building computer models from small pieces.* Paper presented at the Proceedings of the 2007 Summer Computer Simulation Conference, San Diego, CA.

Kaput, J. (1994). Democratizing access to calculus: New routes to old roots. In A. Schoenfeld (Ed.), *Mathematical thinking and problem solving* (pp. 77-156). Hillsdale: Erlbaum.

Kaput, J., Noss, R., & Hoyles, C. (2002). Developing new notations for a learnable mathematics in the computational era. In L. D. English (Ed.), *Handbook of international research in mathematics education*. Mahwah, NJ: Lawrence Erlabum Associates.

Langton, C. G. (1997). *Artificial life: An overview*. Cambridge, MA: The MIT Press.

Levy, S., & Wilensky, U. (2009). Students' learning with the connected chemistry (CC1) curriculum: Navigating the complexities of the particulate world. *Journal of Science Education and Technology, 18*(3), 243-254.

Nemirovsky, R., Tierney, C., & Ogonowski, M. (1993). *Children, additive change, and calculus*. Cambridge, MA: TERC-WP-2-93.

Niss, M., Blum, W., & Galbraith, P. L. (2006). Introduction. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (Eds.), *Modelling and applications in mathematics education: the 14th ICMI study* (pp. 1-32). New York, NY: Springer.

Noss, R., Healy, L., & Hoyles, C. (1997). The construction of mathematical meanings: Connecting the visual with the symbolic. *Educational Studies in Mathematics, 33*, 203-233.

Pozzi, S., Noss, R., & Hoyles, C. (1998). Tools in practice, mathematics in use. *Educational Studies in Mathematics, 36*(2), 105-122.

Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. New York: Basic Books.

Repenning, A., & Ambach, J. (1997). *The agentsheets behavior exchange: Supporting social behavior processing.* Presented at the Conference on Human Factors in Computing Systems.

Roschelle, J., Kaput, J., & Stroup, W. (2000). Simcalc: Accelerating students' engagement with the mathematics of change. In M. Jacobson & R. B. Kozma (Eds.), *Research, design, and implementing advanced technology learning environments*. Hillsdale, NJ: Erlbaum.

Rutherford, F. J., & Ahlgren, A. (1991). *Science for all Americans*. New York: Oxford University Press New York.

Sengupta, P., & Wilensky, U. (2008). On the learnability of electricity as a complex system. In G. Kanselaar, J. van Merriëboer, P. Kirschner & T. de Jong (Eds.), *Proceedings of the Int. Conference for the Learning Sciences,* (Vol. 3, pp. 258-264). Utrecht, The Netherlands: ISLS.

Steed, M. (1992). STELLA, a simulation construction kit: Cognitive processes and educational implications. *Journal of Computers and Mathematics and Science Teaching, 11*, 39-52.

Stroup, W. M. (2002). Understanding qualitative calculus: A structural synthesis of learning research. *International Journal of Computers for Mathematical Learning, 7*, 167-215.

Wilensky, U. (1999). NetLogo: http://ccl.northwestern.edu/netlogo.

Wilensky, U. (2003). Statistical mechanics for secondary school: The gaslab modeling toolkit. *International Journal of Computers for Mathematical Learning, 8*(1), 41.

Wilensky, U. (2006). Complex systems and restructuration of scientific disciplines: Implications for learning, analysis of social systems, and educational policy. In J. Kolodner (Chair) and C. Bereiter & J. Bransford. (Discussants), *Complex Systems, Learning, and Education: Conceptual Principles, Methodologies, and Implications for Educational Research*. Presented at AERA 2006, San Francisco, CA.

Wilensky, U., & Papert, S. (In Preparation). Restructurations: Reformulations of knowledge disciplines through new representational forms.

Wilensky, U., & Rand, W. (in press). *An introduction to agent-based modeling: Modeling natural, social and engineered complex systems with NetLogo.* Cambridge, MA: MIT Press.

Wilensky, U., & Resnick, M. (1999). Thinking in Levels: A Dynamic Systems Perspective to Making Sense of the World. *Journal of Science Education and Technology, 8*(1).

Yerushalmy, M. (1997). Mathematizing verbal descriptions of situations: A language to support modeling. *Cognition and Instruction, 15*(2), 207-264.